Engineering Notes

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A Decision Theory Model for Health Monitoring of Aeroengines

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Nomenclature

x_k = rate of metal liberation or the wear rate in the interval (t_k, t_{k+1})
$\hat{x}_{k|j}$ = estimate of x_k based on the observation record $(y_1, y_2, \ldots y_j)$
t_k = engine hours at the kth oil sample withdrawal
V_k = volume of oil lost in the interval (t_k, t_{k+1}), in liters measured by oil added in the interval
C_k = metal concentration in the kth oil sample measured by spectrophotometer in ppm
m_k = metal carried away by the oil lost in the interval (t_k, t_{k+1})
η_k = process noise sample at t_k
μ_k = measurement noise sample at t_k
y_k = equivalent observation, defined in terms of C_{k+1} and C_k to be used in the Kalman filter formulation
z_k = noise corrupted version of y_k
$E[\cdot]$ = expectation
θ_j, σ_j = mean value and the standard deviation respectively, for the Gaussian distributions corresponding to hypotheses H_j, $j = 1,2$
α_1, α_2 = thresholds for the likelihood ratio test

Introduction

THE Spectrometic Oil Analysis Program (SOAP) is an indirect method of health monitoring of aeroengines. All aeroengines have a forced lubricating system in which lubricating oil is forced past all parts where relative movement in contact exists. Metal particles liberated due to wear are carried away by the oil. They are submicroscopic in nature and their concentrations are of the order of a few ppm under ordinary engine running conditions.[1] In SOAP a sample of lubricating oil is withdrawn from the engine through a drain plug and the concentrations of the desired elements are measured. Engine running hours at each oil withdrawal are also recorded. The measured concentrations have been directly used to represent engine wear. Though large increase in concentration (of two orders of magnitude) indicates increased engine wear, smaller increases in concentration cannot be easily interpreted from observations alone. The health monitoring data of this Note have been obtained using an Atomic Absorption Spectrophotometer (Perkins-Elmer model 403). The spectrometer provides digital readout of various metal concentrations such as iron, copper, and silver in the oil. The data used here are obtained from Rolls-Royce Gnome engines from a Seaking helicopter fleet.

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Under SOAP the volume of lubricating oil in the engine sump and the concentration of various metals in the oil are of prime importance. (The continual loss of oil from the hot-end bearings is periodically replenished and the oil consumption is noted.) A typical set of SOAP data is given in Table 1. In this Note, we develop a decision theoretic model using SOAP data for the health monitoring of aeroengines.[2]

Mathematical Model Development

In this section, a deterministic model and a stochastic model for estimating the metal liberation rate are derived. Results obtained from this deterministic model are currently being used by the operators to obtain a decision regarding the state of the engine health. It is shown that the stochastic model is more suitable for deciding on the engine condition.

The mathematical models for the wear rate of the engine are based on the following assumptions:

1) There is continuous generation of metal due to wear.
2) There is continual loss of engine lubricating oil which carries with it the wear particles held in suspension.
3) Oil loss due to evaporation is negligible.
4) The full volume of oil in the system is equal to the capacity (10 liters).
5) Metal concentration in the lost oil is the average of the concentrations measured at the beginning and end of the interval.
6) The volume of oil added is treated as an error-free measurement while the spectrometric measurements are "noisy."
7) In practice, oil samples are not withdrawn at regular intervals of time. Oil additions in any interval are summed and used as a single addition in the model.
8) Addition of fresh oil in any interval reduces the metal concentration.

A Deterministic Model

The approximate rate of metal liberation in the interval (t_k, t_{k+1}) is given by

$$x_k = [10(C_{k+1} - C_k) + m_k] / (t_{k+1} - t_k) \tag{1}$$

where, on the basis of assumption 5,

$$m_k = V_k (C_k + C_{k+1}) / 2 \tag{2}$$

Table 1 Typical SOAP data, engine number (XXXXX5)

Engine running hours when drawing sample	Oil added to the engine since last sample withdrawal, liters	Concentration in ppm Iron	Copper	Silver
616:15	0.57	0.50	0.20	0.05
627:05	0.57	0.60	0.30	0.0
636:50	1.13	0.80	0.30	0.0
644:05	1.42	0.90	0.30	0.0
653:25	1.42	1.10	not available	0.05
662:55	2.00	1.10	0.30	0.05
673:25	1.00	1.40	0.40	0.10
683:05	1.00	1.30	0.35	3.10

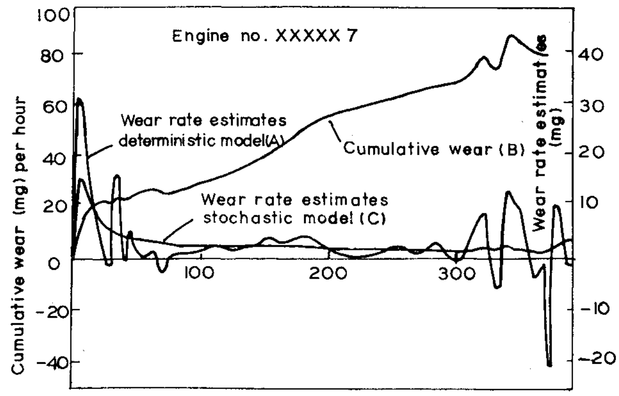

Fig. 1 Estimates of iron wear (engine XXXXX7).

Metal liberation rates from a set of SOAP data calculated using this model, as well as the cumulative metal liberation rate are shown in Fig. 1 (curves A and B). As can be seen from Fig. 1 results obtained from this model do not provide a direct indication to continue or withdraw an engine in service. The inadequacy of this model is due to its deterministic nature.

A Stochastic Model

Optimal estimates of wear $\hat{x}_{k|k}$ can be used for health monitoring of the engine. The estimation scheme has two tasks: 1) to obtain the linear least-square estimates of the wear rate for monitoring purposes and 2) to check if the process indicates any malfunction when estimates show significant deviation. The wear process is represented by the following equation

$$x_{k+1} = x_k + \eta_{k+1} \tag{3}$$

The measurement equation for the spectrometric observations, is

$$z_k = H_k x_k + \mu_k = y_k + \mu_k \tag{4}$$

The measurement noise μ_k, the process noise η_k and x_0 are all assumed to be Gaussian with the following statistics

$$E[\mu_k] = E[\eta_k] = E[x_0] = 0; \qquad E[\mu_k^2] = R; \quad E[\eta_k^2] = Q$$

$$E[x_0^2] = P_0; \quad E[\mu_k \eta_k] = E[x_0 \mu_k] = E[x_0 \eta_k] = 0$$

Metal lost from the oil system in the interval (t_k, t_{k+1}) is given by Eq. (2) in the deterministic model. In the stochastic model, instead of treating C_k as an observation, we calculate an equivalent observation y_k as modeled in Eq. (4). This is obtained by using Eq. (1), wherein m_k is calculated in a more accurate way by

$$m_k = V_k C'_k \tag{5}$$

where C'_k is the effective concentration in the interval (t_k, t_{k+1}) and is given by

$$C'_k = C_k \left(\frac{10 - V_k}{10}\right)\left(\frac{10\,C_k + x_k(t_{k+1} - t_k)}{10\,C_k}\right) \tag{6}$$

The term $(10 - V_k)/10$ is introduced to account for the decrease in metal concentration due to the oil replenishment (assumption 2).

The term $[10\,C_k + x_k(t_{k+1} - t_k)]/10\,C_k$ is introduced to account for the increase in the metal concentration due to the liberation of fresh metal in the interval. From Eqs. (1, 5, and 6),

$$C_{k+1} - \tau_{k+1} C_k = H_k x_k \tag{7}$$

where

$$\tau_{k+1} = \left[1 - \frac{V_k(10 - V_k)}{100}\right]$$

and

$$H_k = \tau_{k+1}(t_{k+1} - t_k)/10$$

Defining, $y_k = C_{k+1} - \tau_{k+1} C_k$; y_k thus becomes a measure of concentration for the engine oil. This new parameter absorbs the irregular time lag between consecutive oil samples and induces stationarity into the measurement equation.

Linear least-squares estimates of engine wear rate can be obtained from the SOAP data using a Kalman filtering scheme[3]

$$\hat{x}_{k|k} = \hat{x}_{k-1|k-1} + K_k(y_k - H_k \hat{x}_{k-1|k-1}) \tag{8a}$$

$$K_k = P_{k|k-1} H_k (P_{k|k-1} H_k^2 + R)^{-1} \tag{8b}$$

$$P_{k|k-1} = P_{k-1|k-1} + Q \tag{8c}$$

$$P_{k|k} = P_{k|k-1}(1 - K_k H_k) \tag{8d}$$

Here $P_{k|k}$ and $P_{k|k-1}$ denote the error covariances and K_k denotes the filter gain. Kalman filters with various values of P_0, Q, and R were used to obtain wear estimates from the SOAP data (Fig. 1 curve C). From a number of filters used, the following conclusions are drawn.[4] 1) Variation of P_0 does not affect the estimates except for the transients with the intial measurements. 2) High values of Q when compared to P_0 result in oscillations in the estimates. The values of Q is chosen such that there are no undue oscillations present in the estimates. 3) The value of R influences the estimates considerably. With small R the estimates fluctuate and enter into the negative region. The value of R is chosen such that the estimates are positive.

Sequential Probability Ratio Test (SPRT) for Malfunction Detection

A Kalman filter provides optimal and reliable estimates when the system is in the normal mode as given by Eq. (3). When the malfunction occurs, Eq. (3) no longer adequately represents the wear process. Precisely when the malfunction occurs, the Kalman filter response is inaccurate as the underlying process model changes. In effect, the failure observations are smoothed by the filter and the ability of the filter to reveal a malfunction diminishes progressively. Since the purpose of health monitoring should be to identify a malfunction as soon as it occurs, we may use an improved filtering scheme based on the innovation sequence and its statistics.[4,5]

Alternatively the malfunction detection problem may be posed as a binary hypothesis testing problem (hypothesis H_1 = the engine is normal and hypothesis H_2 = the engine has a malfunction). The observations $\{y_k\}$, $k = 1,2,\ldots$ are assumed to be from two Gaussian distributions $N_1(\theta_1, \sigma_1^2)$ and $N_2(\theta_2, \sigma_2^2)$ corresponding to the two hypotheses. If the conditional densities corresponding to the two hypotheses are denoted by $P_1(y_1, y_2, \ldots, y_k)$ and $P_2(y_1, y_2, \ldots, y_k)$, the likelihood ratio (LR) becomes

$$LR = \frac{P_2(y_1, y_2, \ldots, y_k)}{P_1(y_1, y_2, \ldots, y_k)} \tag{9}$$

Wald's SPRT is[6]

$LLR \geq \alpha_1$: Decide H_1

$\alpha_2 < LLR < \alpha_1$: Continue to test with additional observations

$LLR \leq \alpha_2$: Decide H_2 (10)

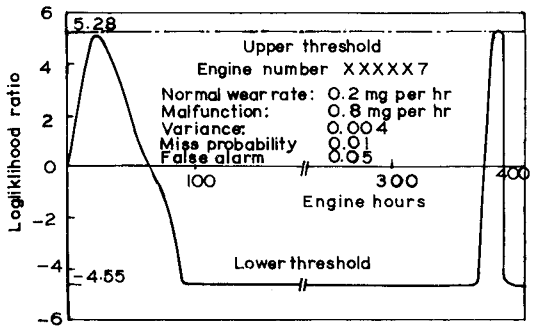

Fig. 2 SPRT based on iron wear estimates.

where *LLR* denotes the log likelihood ratio. For the health monitoring of aeroengines Eqs. (10) may be interpreted as

$LLR \geq \alpha_1$	: Withdraw the engine and recommend for malfunction investigation
$\alpha_2 < LLR < \alpha_1$	: Increase the frequency of oil sampling for SOAP
$LLR \leq \alpha_2$	: Engine "normal" and carry on SOAP at normal frequency

When the *LLR* exceeds the thresholds, the following logic is used. If $LLR \geq \alpha_1$, set $LLR = \alpha_1$. If $LLR \leq \alpha_2$, set $LLR = \alpha_2$. Chien and Adams[7] observed that the detection time is reduced when this technique was used.

Example

The SPRT is conducted on SOAP data obtained from Rolls Royce Gnome engines for a Seaking helicopter fleet. The thresholds α_1 and α_2 can be chosen by considering a criterion function which involves the costs associated with an unforeseen failure and uncalled-for strip examination of the engine, and of testing oil samples at normal and increased frequencies. In the absence of failure statistics, it is difficult to determine the optimum values of α_1 and α_2. From the engine test data the following mean values and variances were assumed for H_1: mean iron liberation rate 0.2 mg/h, variance = 0.004. The mean values and variances corresponding to H_2 are mean iron liberation rate 0.8 mg/h, variance = 0.004. Results of SPRT on the actual data obtained from an aeroengine are given in Fig. 2.

Figure 2 suggests a possible malfunction at about 380 engine hours. For deciding on the engine withdrawal in actual practice, it is desirable to use the SPRT for obtaining wear rate estimates of copper and silver also. The records have shown that the particular engine has indeed been withdrawn at 380 h and a malfunction was detected.

Conclusions

A technique that uses a combination of Kalman filtering and decision theory is presented for the problem of malfunction detection in aeroengines through oil sample analysis. The techniques have obvious potential for use in several other subsystems of an aircraft, for example, control and navigational systems.

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